Eur. Phys. J. A **10**, 109–117 (2001)

THE EUROPEAN PHYSICAL JOURNAL A

Scalar mesons and glueball in a quark model allowing for gluon anomalies

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Received: 11 January 2001 / Revised version: 16 January 2001
Communicated by V.V. Anisovich

Abstract. This paper is a sequel of a previous one (*Scalar mesons in a chiral quark model with glueball*, Eur. Phys. J. A **8**, 567 (2000)) where an attempt to construct an effective $U(3)\times U(3)$-symmetric meson Lagrangian with a scalar glueball was made. The glueball was introduced by using the dilaton model on the base of scale invariance. The scale invariance breaking because of current quark masses and the scale anomaly of QCD, reproduced by the dilaton potential, was taken into account. However, in the previous paper, the scale invariance breaking because of the terms like $h_\phi\phi_0^2$ and $h_\sigma\bar\sigma_0^2$, where ϕ_0 and $\bar\sigma_0$ are the pseudoscalar and scalar isosinglets, was not taken into account. These terms are produced by the part of the 't Hooft interaction that is connected with gluon anomalies. Allowing for the scale invariance breaking by these terms has a decisive effect on the quarkonium-glueball mixing and noticeably changes the widths of glueball strong decays. Taking account of this additional source of the scale invariance breaking and its implications are the subject of the present work. It is also shown that in the decay of a glueball into four pions, the channel with two ρ-resonances dominates.

PACS. 12.39.Ki Relativistic quark model – 12.39.Mk Glueball and nonstandard multiquark/gluon states – 13.25.-k Hadronic decays of mesons – 14.40.-n Mesons

1 Introduction

In our previous work [1], an effective meson Lagrangian including a scalar glueball field was derived from a chiral quark Lagrangian of the Nambu–Jona-Lasinio (NJL) type. The glueball was introduced into the effective meson Lagrangian by using the dilaton model [2]. This allowed us to construct an effective meson Lagrangian which is scale-invariant except for the scale anomaly of QCD reproduced by the dilaton potential and the terms with current quark masses, in accordance with the QCD Lagrangian[1]. However, in [1] we did not take into account another source of the breaking of scale invariance. Indeed, there are terms in the effective Lagrangian that are connected with gluon anomalies and which are produced by the 't Hooft interaction. They describe the singlet-octet mixing among scalar and pseudoscalar mesons and have the following form [3, 4]:

$$L_{\rm an}(\bar\sigma,\phi) = -h_\phi\phi_0^2 + h_\sigma\bar\sigma_0^2, \qquad (1)$$

where ϕ_0 and $\bar\sigma_0$ $(\langle\bar\sigma_0\rangle \neq 0)$ are pseudoscalar and scalar meson isosinglets, respectively, and h_ϕ, h_σ are constants; $\phi_0 = \sqrt{2/3}\phi_u - \sqrt{1/3}\phi_s$, $\bar\sigma_0 = \sqrt{2/3}\bar\sigma_u - \sqrt{1/3}\bar\sigma_s$, where ϕ_u and $\bar\sigma_u$ $(\langle\bar\sigma_u\rangle \neq 0)$ consist of u-quarks and ϕ_s, $\bar\sigma_s$ $(\langle\bar\sigma_s\rangle \neq 0)$ of s-quarks.

When constructing a scale-invariant effective Lagrangian with dilaton fields, these terms require a special treatment taking into account the breaking of scale invariance. Note that the coefficients h_ϕ and h_σ are determined by two different interactions: the 't Hooft interaction and the standard NJL four-quark interaction; and the dilaton field should be inserted into them by using a special prescription (see below sect. 2). Moreover, as can be anticipated, it turns out that these terms determine the most of quarkonia-glueball mixing.

The structure of our paper is the following. In sect. 2, a chiral quark model of the NJL type with the six-quark 't Hooft interaction is bosonized to construct an effective meson Lagrangian. The meson Lagrangian is extended by introducing a scalar glueball as a dilaton on the base of scale invariance. The gap equations, the divergence of the dilatation current and quadratic terms of the meson Lagrangian are derived in sect. 3. The numerical estimates of model parameters are given in sect. 4. In sect. 5, the widths for the main modes of strong decays of scalar isoscalar mesons are calculated. The discussion over the obtained results is given in sect. 6.

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[1] Note that in [1] there was a wrong sign at the last term in formula (43), which led to incorrect estimates for the decay widths of the scalar glueball.

2 Lagrangians

Let us show how the effective meson Lagrangian looks when all three sources of scale invariance breaking mentioned above are taken into account. Recall that the original effective $U(3) \times U(3)$ quark Lagrangian has the following form (see [1]):

$$L = L_{\mathrm{NJL}} + L_{\mathrm{tH}}, \tag{2}$$

$$L_{\mathrm{NJL}} = \bar{q}(i\hat{\partial} - m^0)q + \frac{G}{2}\sum_{a=1}^{9}[(\bar{q}\tau_a q)^2 + (\bar{q}i\gamma_5\tau_a q)^2], \tag{3}$$

$$L_{\mathrm{tH}} = -K\left\{\det[\bar{q}(1+\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]\right\}, \tag{4}$$

where q and $\bar{q}$ stand for u, d, and s quark fields; m^0 is a current quark mass matrix with diagonal elements: m_u^0, m_d^0, m_s^0 ($m_u^0 \approx m_d^0$). The matrices τ_a are related to the Gell-Mann λ_a matrices as follows:

$$\tau_a = \lambda_a \quad (a = 1, ..., 7), \quad \tau_8 = (\sqrt{2}\lambda_0 + \lambda_8)/\sqrt{3},$$
$$\tau_9 = (-\lambda_0 + \sqrt{2}\lambda_8)/\sqrt{3}. \tag{5}$$

Here $\lambda_0 = \sqrt{2/3}\ \mathbf{1}$, with $\mathbf{1}$ being the unit matrix. The term L_{NJL} is the standard $U(3) \times U(3)$-symmetric NJL Lagrangian with four-quark vertices, and L_{tH} is the six-quark 't Hooft interaction.

It is convenient to use an equivalent form of Lagrangian (2) containing only four-quark vertices whose interaction constants take account of the 't Hooft interaction. Using the method described in [1] and [5–7], we obtain

$$L = \bar{q}(i\hat{\partial} - \overline{m}^0)q + \frac{1}{2}\sum_{a,b=1}^{9}[G_{ab}^{(-)}(\bar{q}\tau_a q)(\bar{q}\tau_b q)$$
$$+ G_{ab}^{(+)}(\bar{q}i\gamma_5\tau_a q)(\bar{q}i\gamma_5\tau_b q)], \tag{6}$$

where

$$G_{11}^{(\pm)} = G_{22}^{(\pm)} = G_{33}^{(\pm)} = G \pm 4Km_s I_1^{\Lambda}(m_s),$$
$$G_{44}^{(\pm)} = G_{55}^{(\pm)} = G_{66}^{(\pm)} = G_{77}^{(\pm)} = G \pm 4Km_u I_1^{\Lambda}(m_u),$$
$$G_{88}^{(\pm)} = G \mp 4Km_s I_1^{\Lambda}(m_s), \quad G_{99}^{(\pm)} = G,$$
$$G_{89}^{(\pm)} = G_{98}^{(\pm)} = \pm 4\sqrt{2}Km_u I_1^{\Lambda}(m_u),$$
$$G_{ab}^{(\pm)} = 0 \quad (a \neq b; \quad a, b = 1, \ldots, 7),$$
$$G_{a8}^{(\pm)} = G_{a9}^{(\pm)} = G_{8a}^{(\pm)} = G_{9a}^{(\pm)} = 0 \quad (a = 1, \ldots, 7), \tag{7}$$

and $\bar{m}^0$ is a diagonal matrix composed of modified current quark masses

$$\overline{m}_u^0 = m_u^0 - 32Km_u m_s I_1^{\Lambda}(m_u)I_1^{\Lambda}(m_s), \tag{8}$$

$$\overline{m}_s^0 = m_s^0 - 32Km_u^2 I_1^{\Lambda}(m_u)^2, \tag{9}$$

introduced here to avoid double counting of the 't Hooft interaction in gap equations (see [1,5]). Here m_u and m_s are constituent quark masses and the integrals

$$I_n^{\Lambda}(m_a) = \frac{N_c}{(2\pi)^4}\int \mathrm{d}_e^4 k \frac{\theta(\Lambda^2 - k^2)}{(k^2 + m_a^2)^n}, \tag{10}$$

where $n = 1, 2$ and $a = u, s$, are calculated in the Euclidean metric and regularized by a simple $O(4)$-symmetric ultraviolet cut-off Λ.

After bosonization of Lagrangian (6) and taking into account the spontaneous breaking of chiral symmetry (SBCS) (see, *e.g.*, [1,5]) we obtain

$$\mathcal{L}(\sigma, \phi) = L_{\mathrm{G}}(\sigma, \phi)$$
$$-i\ \mathrm{Tr}\ln\left\{i\hat{\partial} - m + \sum_{a=1}^{9}\tau_a g_a(\sigma_a + i\sqrt{Z}\gamma_5\phi_a)\right\} =$$
$$L_{\mathrm{kin}}(\sigma, \phi) + L_{\mathrm{G}}(\sigma, \phi) + L_{\text{1-loop}}(\sigma, \phi), \tag{11}$$

$$\sigma = \sum_{a=1}^{9}\sigma_a\tau_a, \qquad \phi = \sum_{a=1}^{9}\phi_a\tau_a. \tag{12}$$

Note that $\langle\sigma_u\rangle = \langle\sigma_s\rangle = 0$ ($\sigma_u \equiv \sigma_8$ and $\sigma_s \equiv \sigma_9$) unlike $\bar{\sigma}_u$ and $\bar{\sigma}_s$ introduced after the formula (1) to define $\bar{\sigma}_0$. The fields $\bar{\sigma}_u$ and $\bar{\sigma}_s$ are connected with σ_u and σ_s by the relations

$$\bar{\sigma}_u = \sigma_u - \frac{m_u - \bar{m}_u^0}{g_u}, \qquad \bar{\sigma}_s = \sigma_s + \frac{m_s - \bar{m}_s^0}{\sqrt{2}g_s}, \tag{13}$$

while $\langle\bar{\sigma}_a\rangle = 0$, $(a = 1, \ldots, 7)$.

The term $L_{\mathrm{kin}}(\sigma, \phi)$ contains the kinetic terms

$$L_{\mathrm{kin}}(\sigma, \phi) = \frac{1}{2}\sum_{a=1}^{9}\left((\partial_\nu\sigma_a)^2 + (\partial_\nu\phi_a)^2\right), \tag{14}$$

and the term $L_{\mathrm{G}}(\sigma, \phi)$ is

$$L_{\mathrm{G}}(\sigma, \phi) =$$
$$-\frac{1}{2}\sum_{a,b=1}^{9} g_a\bar{\sigma}_a\left(G^{(-)}\right)_{ab}^{-1} g_b\bar{\sigma}_b$$
$$-\frac{Z}{2}\sum_{a,b=1}^{9} g_a\phi_a\left(G^{(+)}\right)_{ab}^{-1} g_b\phi_b =$$
$$-\frac{1}{2}\sum_{a,b=1}^{9}(g_a\sigma_a - \mu_a + \bar{\mu}_a^0)\left(G^{(-)}\right)_{ab}^{-1}(g_b\sigma_b - \mu_b + \bar{\mu}_b^0)$$
$$-\frac{Z}{2}\sum_{a,b=1}^{9} g_a\phi_a\left(G^{(+)}\right)_{ab}^{-1} g_b\phi_b. \tag{15}$$

Here we introduced, for convenience, the constants μ_a and $\bar{\mu}_a^0$ defined as follows: $\mu_a = 0$, $(a = 1, \ldots, 7)$, $\mu_8 = m_u$, $\mu_9 = -m_s/\sqrt{2}$ and $\bar{\mu}_a^0 = 0$, $(a = 1, \ldots, 7)$, $\bar{\mu}_8^0 = \bar{m}_u^0$, $\bar{\mu}_9^0 = -\bar{m}_s^0/\sqrt{2}$. The term $L_{\text{1-loop}}(\sigma, \phi)$ is the sum of one-loop quark contributions[2]:

$$L_{\text{1-loop}}(\sigma, \phi) = \mathrm{tr}\Bigg[-4mgI_1^{\Lambda}(m)\sigma + 2g^2 I_1^{\Lambda}(m)(\sigma^2 + Z\phi^2)$$
$$+\frac{1}{4}[m, \phi]_-^2 - m^2\sigma^2 + mg\sigma(\sigma^2 + Z\phi^2)$$
$$-\frac{g}{2}[m, \phi]_-[\sigma, \phi]_- - \frac{g^2}{4}((\sigma^2 + Z\phi^2)^2 - [\sigma, \phi]_-^2)\Bigg]. \tag{16}$$

[2] Here we left only the diverging parts of the quark loop diagrams (see [8]).

The Yukawa coupling constants g_a describing the interaction of quarks and mesons appear as a result of renormalization of meson fields (see [1,8] for details):

$$g_1^2 = g_2^2 = g_3^2 = g_8^2 = g_u^2 = [4I_2^\Lambda(m_u)]^{-1},$$
$$g_4^2 = g_5^2 = g_6^2 = g_7^2 = [4I_2^\Lambda(m_u, m_s)]^{-1},$$
$$g_9^2 = g_s^2 = [4I_2^\Lambda(m_s)]^{-1}, \tag{17}$$
$$I_2^\Lambda(m_u, m_s) = \frac{N_c}{(2\pi)^4}\int \mathrm{d}_e^4 k \frac{\theta(\Lambda^2 - k^2)}{(k^2+m_u^2)(k^2+m_s^2)}. \tag{18}$$

For the pseudoscalar meson fields, π-A_1-transitions lead to the factor Z, describing an additional renormalization of pseudoscalar meson fields, with M_{A_1} being the mass of axial-vector meson (see [1,8]):

$$Z = \left(1 - \frac{6m_u}{M_{A_1}^2}\right)^{-1} \approx 1.4. \tag{19}$$

Up to this moment, we just repeated formulae from [1]. Now we begin a discussion about new scheme of the dilaton fields introduction.

According to the prescription described in [1], we introduce the dilaton field into Lagrangian (11) as follows: the dimensional model parameters G, Λ, K, and m_a are replaced by the following rule: $G \to G(\chi_c/\chi)^2$, $K \to K(\chi_c/\chi)^5$, $\Lambda \to \Lambda(\chi/\chi_c)$, $m_a \to m_a(\chi/\chi_c)$, where χ is the dilaton field with the vacuum expectation value χ_c. The current quark masses break scale invariance and, therefore, should not be multiplied by the dilaton field. The modified current quark masses $\bar{m}_a^0$ are also not multiplied by the dilaton field. Finally, we come to the Lagrangian

$$\bar{\mathcal{L}}(\sigma,\phi,\chi) = \mathcal{L}(\chi) + L_{\mathrm{kin}}(\sigma,\phi) + \bar{L}_{\mathrm{G}}(\sigma,\phi,\chi)$$
$$+\bar{L}_{1\text{-loop}}(\sigma,\phi,\chi) + \Delta L_{\mathrm{an}}(\sigma,\phi,\chi). \tag{20}$$

Here $\mathcal{L}(\chi)$ is the pure dilaton Lagrangian

$$\mathcal{L}(\chi) = \frac{1}{2}(\partial_\nu\chi)^2 - V(\chi), \tag{21}$$

with the potential

$$V(\chi) = B\left(\frac{\chi}{\chi_0}\right)^4\left[\ln\left(\frac{\chi}{\chi_0}\right)^4 - 1\right], \tag{22}$$

that has a minimum at $\chi = \chi_0$, and the parameter B representing the vacuum energy when there are no quarks.

Here, the term $\bar{L}_{\mathrm{G}}(\sigma,\phi,\chi)$ is

$$\bar{L}_{\mathrm{G}}(\sigma,\phi,\chi) =$$
$$-\frac{1}{2}\left(\frac{\chi}{\chi_c}\right)^2\sum_{a,b=1}^{9}\left(g_a\sigma_a - \mu_a\frac{\chi}{\chi_c} + \bar{\mu}_a^0\right)\left(G^{(-)}\right)_{ab}^{-1}$$
$$\times\left(g_b\sigma_b - \mu_b\frac{\chi}{\chi_c} + \bar{\mu}_b^0\right)$$
$$-\frac{Z}{2}\left(\frac{\chi}{\chi_c}\right)^2\sum_{a,b=1}^{9} g_a\phi_a\left(G^{(+)}\right)_{ab}^{-1} g_b\phi_b. \tag{23}$$

Expanding (23) in a power series of χ, we can extract a term that is of order χ^4. It can be absorbed by the term in the pure dilaton potential which has the same degree of χ for the reasons given in [1].

The sum of one-loop quark diagrams is denoted as $\bar{L}_{1\text{-loop}}$:

$$\bar{L}_{1\text{-loop}}(\sigma,\phi,\chi) = \mathrm{tr}\left[-4mgI_1^\Lambda(m)\sigma\left(\frac{\chi}{\chi_c}\right)^3\right.$$
$$+2g^2I_1^\Lambda(m)(\sigma^2+Z\phi^2)\left(\frac{\chi}{\chi_c}\right)^2 - m^2g^2\sigma^2\left(\frac{\chi}{\chi_c}\right)^2$$
$$\left.+mg\frac{\chi}{\chi_c}\sigma(\sigma^2+Z\phi^2) - \frac{g^2}{4}(\sigma^2+Z\phi^2)^2\right]. \tag{24}$$

Not that Lagrangian (11) implicitly contains the term L_{an} (see the introduction) that is induced by gluon anomalies. When the procedure of the scale invariance restoration is applied to Lagrangian (11), it also becomes scale invariant. To avoid this, one should subtract this part in the scale-invariant form and add it in a scale-breaking (SB) form. This is achieved by including the term ΔL_{an}:

$$\Delta L_{\mathrm{an}}(\sigma,\phi,\chi) = -L_{\mathrm{an}}(\bar{\sigma},\phi)\left(\frac{\chi}{\chi_c}\right)^2 + L_{\mathrm{an}}^{\mathrm{SB}}(\sigma,\phi,\chi). \tag{25}$$

The term L_{an} was introduced in (1). Let us define the scale-breaking term $L_{\mathrm{an}}^{\mathrm{SB}}$. The coefficients h_σ and h_ϕ in (1) can be determined by comparing them with the terms in (15) that describe the singlet-octet mixing. We obtain

$$h_\phi = -\frac{3}{2\sqrt{2}}g_ug_sZ(G^{(+)})_{89}^{-1}, \quad h_\sigma = \frac{3}{2\sqrt{2}}g_ug_s(G^{(-)})_{89}^{-1}. \tag{26}$$

If these terms were to be made scale-invariant, one should insert $(\chi/\chi_c)^2$ into them (see (25)). However, as the gluon anomalies break scale invariance, we introduce the dilaton field into these terms in a more complicated way. The inverse matrix elements $(G^{(+)})_{ab}^{-1}$ and $(G^{(-)})_{ab}^{-1}$,

$$(G^{(+)})_{89}^{-1} = \frac{-4\sqrt{2}m_uKI_1^\Lambda(m_u)}{G_{88}^{(+)}G_{99}^{(+)} - (G_{89}^{(+)})^2}, \tag{27}$$

$$(G^{(-)})_{89}^{-1} = \frac{4\sqrt{2}m_uKI_1^\Lambda(m_u)}{G_{88}^{(-)}G_{99}^{(-)} - (G_{89}^{(-)})^2}, \tag{28}$$

are determined by two different interactions. The numerators are fully defined by the 't Hooft interaction that leads to anomalous terms (1) breaking scale invariance, therefore, we do not introduce here dilaton fields. The denominators are determined by the constant G describing the standard NJL four-quark interaction, and the dilaton field is inserted into it, according to the prescription given above. Finally, we come to the following form of $L_{\mathrm{an}}^{\mathrm{SB}}$:

$$L_{\mathrm{an}}^{\mathrm{SB}}(\sigma,\phi,\chi) = \left(-h_\phi\phi_0^2 + h_\sigma\left(\sigma_0 - F_0\frac{\chi}{\chi_c} + F_0^0\right)^2\right)\left(\frac{\chi}{\chi_c}\right)^4, \tag{29}$$

$$F_0 = \frac{\sqrt{2}m_u}{\sqrt{3}g_u} + \frac{m_s}{\sqrt{6}g_s}, \qquad F_0^0 = \frac{\sqrt{2}\bar{m}_u^0}{\sqrt{3}g_u} + \frac{\bar{m}_s^0}{\sqrt{6}g_s}. \tag{30}$$

From it, we immediately obtain the term $\Delta L_{\rm an}$:

$$\Delta L_{\rm an} = \left(h_\phi \phi_0^2 - h_\sigma \left(\sigma_0 - F_0 \frac{\chi}{\chi_c} + F_0^0 \right)^2 \right) \times \left(\frac{\chi}{\chi_c} \right)^2 \left(1 - \left(\frac{\chi}{\chi_c} \right)^2 \right). \tag{31}$$

3 Equations

Let us now consider the vacuum expectation value of the divergence of the dilatation current S^μ calculated from the potential of Lagrangian (20):

$$\langle \partial_\mu S^\mu \rangle = \left(\sum_{a=1}^{9} \sigma_a \frac{\partial V}{\partial \sigma_a} + \sum_{a=1}^{9} \phi_a \frac{\partial V}{\partial \phi_a} + \chi \frac{\partial V}{\partial \chi} - 4V \right) \Bigg|_{\substack{\chi=\chi_c \\ \sigma_a=0 \\ \phi_a=0}} = 4B \left(\frac{\chi_c}{\chi_0} \right)^4 - 2h_\sigma \left(F_0 - F_0^0 \right)^2 - \sum_{q=u,d,s} \bar{m}_q^0 \langle \bar{q}q \rangle. \tag{32}$$

Here $V = V(\chi) + \bar{V}(\sigma, \phi, \chi)$, and $\bar{V}(\sigma, \phi, \chi)$ is the potential part of Lagrangian $\bar{\mathcal{L}}(\sigma, \phi, \chi)$ that does not contain the pure dilaton potential. The expression given in (32) is simplified by using the following relation of the quark condensates to integrals $I_1^\Lambda(m_u)$ and $I_1^\Lambda(m_s)$:

$$4m_q I_1^\Lambda(m_q) = -\langle \bar{q}q \rangle_0, \qquad (q = u, d, s), \tag{33}$$

and by taking into account that these integrals are connected with constants $G_{ab}^{(-)}$ through gap equations, as will be shown below (see (39) and (40)). Comparing (32) with the QCD expression

$$\langle \partial_\mu S^\mu \rangle = \mathcal{C}_{\rm g} - \sum_{q=u,d,s} m_q^0 \langle \bar{q}q \rangle, \tag{34}$$

where

$$\mathcal{C}_{\rm g} = \left(\frac{11N_c}{24} - \frac{N_{\rm f}}{12} \right) \left\langle \frac{\alpha}{\pi} \left(G_{\mu\nu}^a \right)^2 \right\rangle, \tag{35}$$

where N_c is the number of colors, $N_{\rm f}$ is the number of flavours, $\langle \frac{\alpha}{\pi} (G_{\mu\nu}^a)^2 \rangle$ and $\langle \bar{q}q \rangle$ are the gluon and quark condensates, one can see that the term $\sum m_q^0 \langle \bar{q}q \rangle$ on the right-hand side of (34) is canceled by the corresponding contribution on the right-hand side of (32). Equating the right hand sides of (32) and (34),

$$\mathcal{C}_{\rm g} - \sum_{q=u,d,s} m_q^0 \langle \bar{q}q \rangle = 4B \left(\frac{\chi_c}{\chi_0} \right)^4 - 2h_\sigma \left(F_0 - F_0^0 \right)^2 - \sum_{q=u,d,s} \bar{m}_q^0 \langle \bar{q}q \rangle, \tag{36}$$

we obtain the correspondence

$$\mathcal{C}_{\rm g} = 4B \left(\frac{\chi_c}{\chi_0} \right)^4 + \sum_{a,b=8}^{9} (\bar{\mu}_a^0 - \mu_a^0)(G^{(-)})_{ab}^{-1}(\mu_b - \bar{\mu}_b^0) - 2h_\sigma \left(F_0 - F_0^0 \right)^2, \tag{37}$$

where $\mu_a^0 = 0$ $(a = 1, \ldots 7)$, $\mu_8^0 = m_u^0$, and $\mu_9^0 = -m_s/\sqrt{2}$. This equation relates the gluon condensate, whose value we take from other models (see, *e.g.*, [9]), to the model parameter B. The next step is to investigate gap equations.

At this step, we introduce the new dilaton field $\chi' = \chi - \chi_c$ with zero vacuum expectation value. In the following calculations, the effective meson Lagrangian is expanded in terms of χ'.

As usual, gap equations follow from the requirement that the terms linear in σ and χ' should be absent in the effective Lagrangian:

$$\frac{\delta \bar{\mathcal{L}}}{\delta \sigma_8} \Bigg|_{\substack{\phi=0 \\ \sigma=0 \\ \chi=\chi_c}} = \frac{\delta \bar{\mathcal{L}}}{\delta \sigma_9} \Bigg|_{\substack{\phi=0 \\ \sigma=0 \\ \chi=\chi_c}} = \frac{\delta \bar{\mathcal{L}}}{\delta \chi} \Bigg|_{\substack{\phi=0 \\ \sigma=0 \\ \chi=\chi_c}} = 0. \tag{38}$$

This leads to the following equations:

$$(m_u - \bar{m}_u^0)(G^{(-)})_{88}^{-1} - \frac{m_s - \bar{m}_s^0}{\sqrt{2}}(G^{(-)})_{89}^{-1} - 8m_u I_1^\Lambda(m_u) = 0, \tag{39}$$

$$(m_s - \bar{m}_s^0)(G^{(-)})_{99}^{-1} - \sqrt{2}(m_u - \bar{m}_u^0)(G^{(-)})_{98}^{-1} - 8m_s I_1^\Lambda(m_s) = 0, \tag{40}$$

$$4B \left(\frac{\chi_c}{\chi_0} \right)^3 \frac{1}{\chi_0} \ln \left(\frac{\chi_c}{\chi_0} \right)^4 + \frac{1}{\chi_c} \sum_{a,b=8}^{9} \bar{\mu}_a^0 (G^{(-)})_{ab}^{-1} (\bar{\mu}_b^0 - 3\mu_b) - \frac{2h_\sigma}{\chi_c} \left(F_0 - F_0^0 \right)^2 = 0. \tag{41}$$

Using (8) and (9), one can rewrite eqs. (39) and (40) in the well-known form [7]:

$$m_u^0 = m_u - 8Gm_u I_1^\Lambda(m_u) - 32Km_u m_s I_1^\Lambda(m_u) I_1^\Lambda(m_s), \tag{42}$$

$$m_s^0 = m_s - 8Gm_s I_1^\Lambda(m_s) - 32K(m_u I_1^\Lambda(m_u))^2. \tag{43}$$

To determine the masses of quarkonia and of the glueball, let us consider the part of Lagrangian (20) which is quadratic in fields σ and χ' and which is denoted as $L^{(2)}$

$$L^{(2)}(\sigma, \phi, \chi') = -\frac{1}{2} g_u^2 \{ [(G^{(-)})_{88}^{-1} - 8I_1^\Lambda(m_u)] + 4m_u^2 \} \sigma_u^2 - \frac{1}{2} g_s^2 \{ [(G^{(-)})_{99}^{-1} - 8I_1^\Lambda(m_s)] + 4m_s^2 \} \sigma_s^2 - g_u g_s (G^{(-)})_{89}^{-1} \sigma_u \sigma_s - \frac{M_{\rm g}^2 \chi'^2}{2} + \sum_{a,b=8}^{9} \frac{\bar{\mu}_a^0}{\chi_c} (G^{(-)})_{ab}^{-1} g_b \sigma_b \chi' + \frac{4h_\sigma (F_0 - F_0^0)}{\chi_c \sqrt{3}} \left(\sigma_s - \sigma_u \sqrt{2} \right) \chi', \tag{44}$$

Table 1. The masses of physical scalar meson states $\sigma_{\rm I}$, $\sigma_{\rm II}$, $\sigma_{\rm III}$ and the values of the parameters χ_c, χ_0, bag constant B, and (bare) glueball mass $M_{\rm g}$ (in MeV) for two cases: 1) $M_{\sigma_{\rm III}} =$ 1500 MeV and 2) $M_{\sigma_{\rm III}} = 1710$ MeV.

	$\sigma_{\rm I}$	$\sigma_{\rm II}$	$\sigma_{\rm III}$	χ_c	χ_0	B(GeV^4)	$M_{\rm g}$
I	400	1100	1500	206	190	0.009	1447
II	400	1100	1710	180	166	0.009	1665

where

$$M^2_{\rm g} = \frac{1}{\chi_c^2}\Bigg(4\mathcal{C}_{\rm g} + \sum_{a,b=8}^{9} \bar\mu^0_a (G^{(-)})^{-1}_{ab}(2\bar\mu^0_b - \mu_b) + \sum_{a,b=8}^{9} 4\mu^0_a (G^{(-)})^{-1}_{ab}(\mu_b - \bar\mu^0_b) - h_\sigma 4F_0^2 + 4h_\sigma (F^0_0)^2\Bigg) \quad (45)$$

is the glueball mass before taking account of mixing effects. Here, the gap equations and eq. (37) are taken into account.

From this Lagrangian, after diagonalization, we obtain the masses of three scalar meson states: $\sigma_{\rm I}$, $\sigma_{\rm II}$, and $\sigma_{\rm III}$, and a matrix of mixing coefficients b that connects the nondiagonalized fields $\sigma_u, \sigma_s, \chi'$ with the physical ones $\sigma_{\rm I}, \sigma_{\rm II}, \sigma_{\rm III}$:

$$\begin{pmatrix}\sigma_u\\ \sigma_s\\ \chi'\end{pmatrix} = \begin{pmatrix} b_{\sigma_u\sigma_{\rm I}} & b_{\sigma_u\sigma_{\rm II}} & b_{\sigma_u\sigma_{\rm III}}\\ b_{\sigma_s\sigma_{\rm I}} & b_{\sigma_s\sigma_{\rm II}} & b_{\sigma_s\sigma_{\rm III}}\\ b_{\chi'\sigma_{\rm I}} & b_{\chi'\sigma_{\rm II}} & b_{\chi'\sigma_{\rm III}}\end{pmatrix}\begin{pmatrix}\sigma_{\rm I}\\ \sigma_{\rm II}\\ \sigma_{\rm III}\end{pmatrix}. \quad (46)$$

4 Model parameters and numerical estimates

The basic parameters of our model are G, K, Λ, m_u, and m_s. After the dilaton fields are introduced, they keep their values [5]:

$$m_u = 280 \text{ MeV}, \quad m_s = 420 \text{ MeV}, \quad \Lambda = 1.26 \text{ GeV}, \quad G = 4.38 \text{ GeV}^{-2}, \quad K = 11.2 \text{ GeV}^{-5}. \quad (47)$$

Moreover, new three parameters χ_0, χ_c, and B appear. To fix the new parameters, one should use eqs. (37), (41), and the physical glueball mass. As a result, we obtain for χ_0 and B:

$$\chi_0 = \chi_c \exp\Bigg\{-\Bigg[\sum_{a,b=8}^{9} \bar\mu^0_a (G^{(-)})^{-1}_{ab}(3\mu_b - \bar\mu^0_b) + 2h_\sigma\left(F_0 - F^0_0\right)^2\Bigg]\Bigg/4\Bigg[\mathcal{C}_{\rm g} - (\bar\mu^0_a - \mu^0_a)(G^{(-)})^{-1}_{ab}(\mu_b - \bar\mu^0_b) + 2h_\sigma\left(F_0 - F^0_0\right)^2\Bigg]\Bigg\}, \quad (48)$$

$$B = \frac{1}{4}\Bigg(\mathcal{C}_{\rm g} - (\bar\mu^0_a - \mu^0_a)(G^{(-)})^{-1}_{ab}(\mu_b - \bar\mu^0_b) + 2h_\sigma\left(F_0 - F^0_0\right)^2\Bigg)\left(\frac{\chi_0}{\chi_c}\right)^4. \quad (49)$$

Table 2. Elements of the matrix b, describing mixing in the scalar isoscalar sector. The upper table refers to the case $\sigma_{\rm III} \equiv f_0(1500)$, the lower one to the case $\sigma_{\rm III} \equiv f_0(1710)$.

	$\sigma_{\rm I}$	$\sigma_{\rm II}$	$\sigma_{\rm III}$
σ_u	0.939	0.240	0.247
σ_s	−0.214	0.968	−0.128
χ'	−0.270	0.067	0.960

	$\sigma_{\rm I}$	$\sigma_{\rm II}$	$\sigma_{\rm III}$
σ_u	0.948	0.232	0.216
σ_s	−0.216	0.971	−0.099
χ'	−0.233	0.047	0.971

We adjust the parameter χ_c so that the mass of the heaviest scalar meson, $\sigma_{\rm III}$, would be either 1500 MeV or 1710 MeV. The result of our fit for both cases is given in table 1. One will also find the mixing coefficients in table 2.

5 Strong decays of scalar mesons

Once all parameters are fixed, we can estimate the decay widths for the main strong decay modes of scalar mesons: $\sigma_l \to \pi\pi, KK$, $\eta\eta$, $\eta\eta'$, and 4π where $l = {\rm I}, {\rm II}, {\rm III}$.

Note that, in the energy region under consideration ($\sim$ 1500 MeV), we work on the brim of the validity of exploiting the chiral symmetry that was used to construct our effective Lagrangian. Thus, we can consider our results as rather qualitative.

The vertices describing meson decays can be taken from Lagrangian (20). Below we display only those necessary to calculate the widths of the decays under consideration:

$$L^{(3)} = L^{(3)}_{\rm gl} + L^{(3)}_{\rm q} + L^{(3)}_{\rm an}, \quad (50)$$

$$\begin{aligned} L^{(3)}_{\rm gl} &= A^g_{\pi\pi}\chi'(2\pi^+\pi^- + \pi^0\pi^0) \\ &+ A^g_{KK}\chi'(K^+K^- + K^0\tilde K^0) \\ &+ A^g_{\eta\eta}\chi'\eta\eta + A^g_{\sigma\sigma}\chi'\sigma\sigma, \end{aligned} \quad (51)$$

$$\begin{aligned} L^{(3)}_{\rm q} &= A^u\sigma_u(2\pi^+\pi^- + \pi^0\pi^0) \\ &+ A^u_{KK}\sigma_u(K^+K^- + K^0\tilde K^0) \\ &+ A^s_{KK}\sigma_s(K^+K^- + K^0\tilde K^0) + A^u\sin\bar\theta^2\sigma_u\eta\eta \\ &+ A^s\cos\bar\theta^2\sigma_s\eta\eta - A^u\sin 2\bar\theta\sigma_u\eta\eta' \\ &+ A^s\sin 2\bar\theta\sigma_s\eta\eta' + A^u Z^{-1}\sigma_u^3, \end{aligned} \quad (52)$$

$$\begin{aligned} L^{(3)}_{\rm an} &= A^{\rm an}_\phi \sin^2\theta\chi'\eta\eta - A^{\rm an}_\phi\sin 2\theta\chi'\eta\eta' \\ &+ A^{\rm an}_\sigma\chi'\sigma_u\sigma_u, \end{aligned} \quad (53)$$

where $L^{(3)}_{\rm gl}$, $L^{(3)}_{\rm q}$, and $L^{(3)}_{\rm an}$ contain the vertices describing decays of the pure glueball, pure quarkonia, and the anomaly induced vertices describing pure glueball decays, respectively.

The constants at the vertices in (51)–(53) are defined as follows:

$$A^g_{\pi\pi} = -\frac{M^2_\pi}{\chi_c}, \qquad A^g_{KK} = -\frac{2M^2_K}{\chi_c},$$

$$A^g_{\eta\eta} = -\frac{M^2_\eta}{\chi_c}, \qquad A^g_{\sigma\sigma} = -\frac{M^2_{\sigma_u}}{\chi_c}, \tag{54}$$

$$A^u = 2g_u m_u Z, \qquad A^s = -2\sqrt{2} g_s m_s Z,$$

$$A^u_{KK} = 2g_u Z \left(\frac{m_u + m_s}{2} \left(\frac{F_\pi}{F_K} \right)^2 + \frac{m_s(m_u - m_s)}{m_u + m_s} \right),$$

$$A^s_{KK} = -2\sqrt{2} g_s Z \left(\frac{m_u + m_s}{2} \left(\frac{F_s}{F_K} \right)^2 + \frac{m_u(m_s - m_u)}{m_u + m_s} \right), \tag{55}$$

$$A^{\mathrm{an}}_\phi = -\frac{2h_\phi}{\chi_c}, \qquad A^{\mathrm{an}}_\sigma = \frac{2h_\sigma}{3\chi_c}, \tag{56}$$

where $\bar\theta = \theta - \theta_0$, with θ being the singlet-octet mixing angle in the pseudoscalar channel, $\theta \approx -19°$ [8], and θ_0 the ideal mixing angle, $\tan\theta_0 = 1/\sqrt{2}$. The pion and kaon weak decay constants are denoted as F_π and F_K, respectively, and $F_s = m_s/(g_s\sqrt{Z})$ (see [8]).

Let us start with the lightest scalar isoscalar meson state σ_{I}, associated with f_0(400-1200). This state decays into pions. This is the only strong decay mode, because σ_{I} is too light for other channels to be open. The amplitude describing its decay into pions has the form:

$$A_{\sigma_{\mathrm{I}} \to \pi^+\pi^-} = 2A^g_{\pi\pi} b_{\chi'\sigma_{\mathrm{I}}} + 2A^u b_{\sigma_u \sigma_{\mathrm{I}}}, \tag{57}$$

where the coefficients $b_{\chi'\sigma_{\mathrm{I}}}$ and $b_{\sigma_u\sigma_{\mathrm{I}}}$ represent the corresponding elements of the 3×3 mixing matrix for scalar isoscalar states (see table 2). The glueball and quarkonium contributions have equal signs and increase the width of σ_{I}.

The amplitude (57) leads to the following width of σ_{I}:

$$\Gamma_{\sigma_{\mathrm{I}} \to \pi\pi} = \frac{3}{2} \Gamma_{\sigma_{\mathrm{I}} \to \pi^+\pi^-} \approx 820 \text{ MeV}, \tag{58}$$

for σ_{III} identified with $f_0(1500)$, and

$$\Gamma_{\sigma_{\mathrm{I}} \to \pi\pi} \approx 830 \text{ MeV}, \tag{59}$$

for the case $\sigma_{\mathrm{III}} \equiv f_0(1710)$. The experimental value is known with a large uncertainty and is reported to lie in the interval from 600 to 1000 MeV [10].

The amplitude describing the decay of the state σ_{II} that we identify with $f_0(980)$ into pions also consists of two parts

$$A_{\sigma_{\mathrm{II}} \to \pi^+\pi^-} = 2A^g_{\pi\pi} b_{\chi'\sigma_{\mathrm{II}}} + 2A^u b_{\sigma_u \sigma_{\mathrm{II}}}. \tag{60}$$

Here the glueball contribution is small again and the quarkonium determines the decay width, however, in this case both contributions are opposite in sign and slightly compensate each other. For the decay width, we obtain

$$\Gamma_{\sigma_{\mathrm{II}} \to \pi\pi} \approx 28 \text{ MeV}, \tag{61}$$

if $\sigma_{\mathrm{III}} \equiv f_0(1500)$ and

$$\Gamma_{\sigma_{\mathrm{II}} \to \pi\pi} \approx 26 \text{ MeV}, \tag{62}$$

if $\sigma_{\mathrm{III}} \equiv f_0(1710)$. The experiment gives for the decay of σ_{II} into pions a value lying within the range 30–70 MeV [10].

Now let us proceed with decays of σ_{III}. The process $\sigma_{\mathrm{III}} \to \pi^+\pi^-$ is given by the amplitude

$$A_{\sigma_{\mathrm{III}} \to \pi^+\pi^-} = 2A^g_{\pi\pi} b_{\chi'\sigma_{\mathrm{III}}} + 2A^u b_{\sigma_u \sigma_{\mathrm{III}}} \tag{63}$$

that consists of two parts. The first part represents the contribution from the pure glueball. This contribution is small (since it is proportional to the pion mass squared), and the process is determined by the second part that describes the decay of the quarkonium component. As a result, the width of the decay $\sigma_{\mathrm{III}} \to \pi\pi$ if $\sigma_{\mathrm{III}} \equiv f_0(1500)$ is

$$\Gamma_{\sigma_{\mathrm{III}} \to \pi\pi} \approx 14 \text{ MeV}, \tag{64}$$

and, if $\sigma_{\mathrm{III}} \equiv f_0(1710)$,

$$\Gamma_{\sigma_{\mathrm{III}} \to \pi\pi} \approx 8 \text{ MeV}. \tag{65}$$

In the case of $K\bar{K}$ channels, the contribution of the pure glueball is also proportional to the kaon mass squared, and is rather large as compared to the pions case. The amplitude of the decay $\sigma_{\mathrm{III}} \to K^+K^-$ consists of three parts:

$$A_{\sigma_{\mathrm{III}} \to K^+K^-} = A^g_{KK} b_{\chi'\sigma_{\mathrm{III}}} + A^u_{KK} b_{\sigma_u \sigma_{\mathrm{III}}} + A^s_{KK} b_{\sigma_s \sigma_{\mathrm{III}}}. \tag{66}$$

In the case when σ_{III} is $f_0(1500)$, we have

$$\Gamma_{\sigma_{\mathrm{III}} \to K\bar{K}} = 2\Gamma_{\sigma_{\mathrm{III}} \to K^+K^-} \approx 29 \text{ MeV}, \tag{67}$$

and in the other case ($\sigma_{\mathrm{III}} \equiv f_0(1710)$)

$$\Gamma_{\sigma_{\mathrm{III}} \to K\bar{K}} \approx 60 \text{ MeV}. \tag{68}$$

The amplitude of the decay of σ_{III} into $\eta\eta$ and $\eta\eta'$ can also be considered in the same manner. The only complication is the singlet-octet mixing in the pseudoscalar sector and additional vertices coming from ΔL_{an}. The corresponding amplitude of the decay into $\eta\eta$ is

$$A_{\sigma_{\mathrm{III}} \to \eta\eta} = 2A^g_{\eta\eta} b_{\chi'\sigma_{\mathrm{III}}} + 2A^u \sin^2\bar\theta b_{\sigma_u \sigma_{\mathrm{III}}} + 2A^s \cos^2\bar\theta b_{\sigma_s \sigma_{\mathrm{III}}} + 2A^{\mathrm{an}}_\phi \sin^2\theta b_{\chi'\sigma_{\mathrm{III}}}. \tag{69}$$

The decay widths thereby are, if $\sigma_{\mathrm{III}} \equiv f_0(1500)$,

$$\Gamma_{\sigma_{\mathrm{III}} \to \eta\eta} \approx 25 \text{ MeV}, \tag{70}$$

and, if $\sigma_{\mathrm{III}} \equiv f_0(1710)$,

$$\Gamma_{\sigma_{\mathrm{III}} \to \eta\eta} \approx 43 \text{ MeV}. \tag{71}$$

For the decay of σ_{III} into $\eta\eta'$, we have the following amplitude:

$$A_{\sigma_{\mathrm{III}} \to \eta\eta'} = -A^u \sin 2\bar\theta b_{\sigma_u \sigma_{\mathrm{III}}} + A^s \sin 2\bar\theta b_{\sigma_s \sigma_{\mathrm{III}}} - A^{\mathrm{an}}_\phi \sin 2\theta b_{\chi'\sigma_{\mathrm{III}}}. \tag{72}$$

The direct decay of a bare glueball into $\eta\eta'$ is absent here. The process occurs only due to the mixing of the glueball and scalar isoscalar quarkonia and the anomaly contribution. The decay widths are as follows:

$$\Gamma_{\sigma_{\rm III}\to\eta\eta'} \sim 10\ {\rm MeV}\,, \quad (73)$$

for $\sigma_{\rm III} \equiv f_0(1500)$, and

$$\Gamma_{\sigma_{\rm III}\to\eta\eta'} \approx 30\ {\rm MeV}\,, \quad (74)$$

for $\sigma_{\rm III} \equiv f_0(1710)$. The estimate for the decay $f_0(1500)$ into $\eta\eta'$ is very rough, because the decay is allowed only due to a finite width of the resonance as its mass lies a little bit below the $\eta\eta'$ threshold. The calculation is made for the mass of $f_0(1500)$ plus its half-width. For $f_0(1710)$, we have a more reliable estimate, since its mass is large enough for the decay to be possible.

Up to this moment we considered only decays into a pair of mesons. For the state $\sigma_{\rm III}$, there is a possibility to decay into 4 pions. This decay can occur through intermediate σ (f_0(400-1200)) resonance.

The decay through the σ-resonances can be represented as two processes: with two resonances $\sigma_{\rm III}\to\sigma\sigma\to 4\pi$ and one resonance $\sigma_{\rm III}\to\sigma 2\pi\to 4\pi$. The decay of a glueball into two σ is given by the amplitude

$$A_{\sigma_{\rm III}\to\sigma\sigma} \approx 2A^g_{\sigma\sigma} b_{\chi'\sigma_{\rm III}} + 3Z^{-1}A^u b_{\sigma_u\sigma_{\rm III}} b_{\sigma_u\sigma_I} b_{\sigma_u\sigma_{\rm I}} + 2A^{\rm an}_{\sigma} b_{\chi'\sigma_{\rm III}} b^2_{\sigma_u\sigma_{\rm I}}. \quad (75)$$

The amplitude describing the decay into $2\pi^+2\pi^-$ through two σ-resonances is

$$A_{\sigma_{\rm III}\to\sigma\sigma\to 2\pi^+2\pi^-} = 2A_{\sigma_{\rm III}\to\sigma\sigma} A^2_{\sigma\to\pi^+\pi^-} \times(\Delta_\sigma(s_{12})\Delta_\sigma(s_{34}) + \Delta(s_{14})\Delta(s_{23})), \quad (76)$$

where the function $\Delta_\sigma(s)$ appears due to the resonant structure of the processes

$$\Delta_\sigma(s) = (s - M^2_{\sigma_{\rm I}} + iM_{\sigma_{\rm I}}\Gamma_{\sigma_{\rm I}})^{-1}, \quad (77)$$

where $\Gamma_{\sigma_{\rm I}}$ is the decay width of the $\sigma_{\rm I}$ resonance (see (58) and (59)). This function depends on an invariant mass squared s_{ij} defined as follows:

$$s_{ij} = (k_i + k_j)^2, \qquad (i,j = 1,\ldots,4). \quad (78)$$

Here i and j enumerate the momenta k_i of pions $\pi^+(k_1)$, $\pi^-(k_2)$, $\pi^+(k_3)$, and $\pi^-(k_4)$.

Now let us consider the decay into 4π through one σ-resonance. The process is described by two vertices in Lagrangian (20):

$$A^g_{\sigma 2\pi}\chi'\sigma_u(2\pi^+\pi^- + \pi^0\pi^0) + A^u_{\sigma 2\pi}\sigma_u\sigma_u(2\pi^+\pi^- + \pi^0\pi^0), \quad (79)$$

and

$$A^u_{\sigma_{\rm III}\to\sigma 2\pi} = -g_u^2 Z, \quad A^g_{\sigma 2\pi} = \frac{2m_u g_u Z}{\chi_c} \quad (80)$$

are the glueball and quarkonia amplitudes. Thus, the amplitude describing this process is as follows:

$$A_{\sigma_{\rm III}\to\sigma 2\pi} = 2A^g_{\sigma 2\pi}(b_{\sigma_u\sigma_{\rm I}} b_{\chi'\sigma_{\rm III}} + b_{\sigma_u\sigma_{\rm III}} b_{\chi'\sigma_{\rm I}}) + 4A^u_{\sigma 2\pi} b_{\sigma_u\sigma_{\rm III}} b_{\sigma_u\sigma_{\rm I}}. \quad (81)$$

The glueball contribution prevails over the quarkonium one in magnitude and is opposite in sign.

The amplitude describing the decay $\sigma_{\rm III}\to 2\pi^+2\pi^-$ through one σ-resonance is

$$A_{\sigma_{\rm III}\to\sigma 2\pi\to 2\pi^+2\pi^-} = -A_{\sigma_{\rm III}\to\sigma 2\pi} A_{\sigma\to\pi^+\pi^-} \times(\Delta_\sigma(s_{12}) + \Delta_\sigma(s_{34}) + \Delta_\sigma(s_{14}) + \Delta_\sigma(s_{23}))\,. \quad (82)$$

The amplitudes for the decays into $2\pi^0\pi^+\pi^-$ and $4\pi^0$ are calculated in a similar way (see [11] for details). As a result, we obtain for $\sigma_{\rm III} \equiv f_0(1500)$ the following decay widths:

$$\Gamma_{\sigma_{\rm III}\to 2\pi^+2\pi^-} \approx 2.2\,{\rm MeV}, \quad \Gamma_{\sigma_{\rm III}\to 2\pi^0\pi^+\pi^-} \approx 1.2\,{\rm MeV}, \quad \Gamma_{\sigma_{\rm III}\to 4\pi^0} \approx 0.1\,{\rm MeV}. \quad (83)$$

The total width of $\sigma_{\rm III}$ is, therefore, $\Gamma^{\rm tot}_{\sigma_{\rm III}\to 4\pi} \approx 3.5\,{\rm MeV}$. In the other case ($\sigma_{\rm III} \equiv f_0(1710)$),

$$\Gamma_{\sigma_{\rm III}\to 2\pi^+2\pi^-} \approx 6\,{\rm MeV}, \quad \Gamma_{\sigma_{\rm III}\to 2\pi^0\pi^+\pi^-} \approx 3.3\,{\rm MeV}, \quad \Gamma_{\sigma_{\rm III}\to 4\pi^0} \approx 0.3\,{\rm MeV}, \quad (84)$$

and the total width is $\Gamma^{\rm tot}_{\sigma_{\rm III}\to 4\pi} \approx 10\,{\rm MeV}$. As one can see, these values are very small. This is a result of strong compensations between the glueball and quarkonia contributions.

The other possibility of the state $\sigma_{\rm III}$ to decay into 4 pions is to produce two intermediate ρ-resonances ($\sigma_{\rm III}\to\rho\rho\to 4\pi$). Contrary to the decay through scalar resonances, where strong compensations take place, in the process with ρ-resonances, no compensation occurs, and it turns out that the decay through ρ determines the most part of the decay width of $\sigma_{\rm III}$.

To calculate the amplitude describing the process $\sigma_{\rm III}\to 2\rho$, we need a piece of the Lagrangian with ρ-meson fields. Although we did not consider vector mesons in the source Lagrangian, an extended version of NJL model [8, 12] contains the vector and axial-vector fields. Taking the mass term for ρ-mesons from [8,12] and including dilaton fields into it according to the principle of scale invariance, we obtain

$$\frac{M^2_\rho}{2}\left(\frac{\chi}{\chi_c}\right)^2(2\rho^+_\mu\rho^-_\mu + \rho^0_\mu\rho^0_\mu), \quad (85)$$

where $M_\rho = 770$ MeV is the ρ-meson mass. From this, we derive the vertex describing the decay $\sigma_{\rm III}\to\rho\rho$:

$$\frac{M^2_\rho}{\chi_c} b_{\chi'\sigma_{\rm III}}\chi'(2\rho^+_\mu\rho^-_\mu + \rho^0_\mu\rho^0_\mu). \quad (86)$$

The decay of a ρ-meson into pions is described by the following amplitude:

$$g_\rho(p_1 - p_2)^\mu\,, \quad (87)$$

where $g_\rho = 6.14$ is the ρ-meson decay constant, p_1 and p_2 are the momenta of π^+ and π^-. Finally, we come to the

Table 3. The partial and total decay widths (in MeV) of the scalar meson states f_0(400-1200), f_0(980) and of the glueball for two cases: $\sigma_{\rm III} \equiv f_0(1500)$ and $\sigma_{\rm III} \equiv f_0(1710)$, and experimental values of decay widths of f_0(1500) and f_0(1710) [10].

	f_0(400-1200)	f_0(980)	f_0(1500)	f_0(1710)
$\Gamma_{\pi\pi}$	820	28	14	8
$\Gamma_{K\bar{K}}$	–	–	29	60
$\Gamma_{\eta\eta}$	–	–	25	43
$\Gamma_{\eta\eta'}$	–	–	$\sim$ 10	30
$\Gamma_{4\pi}$	–	–	140	$\sim$ 1000
$\Gamma_{\rm tot}$	820	28	220	$\sim$ 1100
$\Gamma_{\rm tot}^{\rm exp}$	600–1200	40–100	112	130

following formula for the amplitude of the process $\sigma_{\rm III} \to \rho^0\rho^0 \to 2\pi^+2\pi^-$:

$$A_{\sigma_{\rm III}\to\rho^0\rho^0\to 2\pi^+2\pi^-} = \frac{M_\rho^2 g_\rho^2 b_{\chi'\sigma_{\rm III}}}{\chi_c} \times\Big((s_{13}+s_{24}-s_{14}-s_{23})\Delta_\rho(s_{12})\Delta_\rho(s_{34}) +(s_{13}+s_{24}-s_{12}-s_{34})\Delta_\rho(s_{14})\Delta_\rho(s_{23})\Big). \quad (88)$$

The function $\Delta_\rho(s)$ is the following:

$$\Delta_\rho(s) = (s - M_\rho^2 + iM_\rho\Gamma_\rho)^{-1}. \quad (89)$$

Here $\Gamma_\rho = 150$ MeV is the decay width of the ρ-resonance. The decay into $2\pi^0\pi^+\pi^-$ occurs through a pair of charged ρ-resonances: ρ^+ and ρ^-. The amplitude of this process is the same as for the decay with intermediate ρ^0. The decay into $4\pi^0$ cannot go via ρ-resonances.

In an extended NJL model [8,12], there are no vertices describing the decay of a quarkonium into ρ-mesons. As a result, only the glueball part determines the decay of $\sigma_{\rm III}$ into 4 pions through ρ-resonances unlike the case with σ resonances. This leads to a large decay rate through ρ-mesons (contrary to decays through σ). As a result, we obtain for the decay widths of $\sigma \to 4\pi$ via ρ-resonances if $\sigma_{\rm III} \equiv f_0(1500)$:

$$\Gamma_{\sigma_{\rm III}\to\rho\rho\to 2\pi^+2\pi^-} \approx 50\,\text{MeV},\ \Gamma_{\sigma_{\rm III}\to\rho\rho\to 2\pi^0\pi^+\pi^-} \approx 90\,\text{MeV}, \quad (90)$$

with the total width $\Gamma_{\sigma_{\rm III}\to 4\pi}^{\rm tot} \approx 140\,$MeV. In the other case ($\sigma_{\rm III} \equiv f_0(1710)$),

$$\Gamma_{\sigma_{\rm III}\to\rho\rho\to 2\pi^+2\pi^-} \approx 350\,\text{MeV},$$
$$\Gamma_{\sigma_{\rm III}\to\rho\rho\to 2\pi^0\pi^+\pi^-} \approx 650\ \text{MeV}, \quad (91)$$

and the total width $\Gamma_{\sigma_{\rm III}\to 4\pi}^{\rm tot} \approx 1$GeV.

Now we can estimate the total width of the state $\sigma_{\rm III}$. If $\sigma_{\rm III}$ is identified with $f_0(1500)$, we have

$$\Gamma_{\sigma_{\rm III}}^{\rm tot} \approx 220\ \text{MeV}, \quad (92)$$

which is in qualitative agreement with the experimental value 112 MeV [10], and, in the other case ($\sigma_{\rm III} \equiv f_0(1710)$)

$$\Gamma_{\sigma_{\rm III}}^{\rm tot} \approx 1.2\ \text{GeV}, \quad (93)$$

exceeding the experimental value 130 MeV [10] by an order. In the last case ($f_0(1710)$), ρ-mesons can show up as on-mass-shell decay products at large probability. The decay width is estimated as ∼1 GeV. The absence of this decay mode in experimental observations is a reason that $f_0(1710)$ is not a glueball.

Our estimates for the decay widths of the scalar meson states $\sigma_{\rm I}$, $\sigma_{\rm II}$, and $\sigma_{\rm III}$ are collected in table 3.

6 Conclusion

In the approach presented here, we assume that (with the exception of the dilaton potential and the ’t Hooft interaction) scale invariance holds for the effective Lagrangian before and after SBCS in the chiral limit. On the other hand, we take into account the effects of scale invariance breaking that come from three sources: the terms with current quark masses, the dilaton potential reproducing the scale anomaly of QCD, and term $L_{\rm an}$ induced by gluon anomalies (see (1) in the introduction).

The scale invariance breaking that is connected with the term $L_{\rm an}$ was not taken into account in our previous paper [1]. This led to a small quarkonia-glueball mixing proportional to current quark masses, disappearing in the chiral limit. If the term $\Delta L_{\rm an}$ is taken into account in (20), the quarkonia-glueball mixing becomes much greater and does not disappear in the chiral limit, being proportional to constituent quark masses (quark condensates). This accords to the results obtained from QCD in [13]. This contribution to the quarkonia-glueball mixing turns out to have decisive effect on the strong decay widths of scalar mesons.

For the scalar meson states f_0(400-1200) and f_0(980), we obtain good agreement with experimental data [10]. Their decay widths are determined by quarkonium parts of decay amplitudes.

Strong decays of the scalar meson state $\sigma_{\rm III}$ (“glueball”) are considered for two different masses: 1500 MeV and 1710 MeV. In the $\pi\pi$ channel, the contribution from quarkonia prevails over that from the glueball and thereby determines the decay rate. In the case of KK, $\eta\eta$, $\eta\eta'$ channels, there are noticeable compensations among different parts of the decay amplitudes.

A similar situation with compensations takes place in the decay of a glueball into 4π with intermediate σ-mesons. Here we have a strong compensation between the glueball and quarkonia contributions. But there is a possibility for the state $\sigma_{\rm III}$ to decay through ρ-resonances. In this case, as the quarkonium component is absent, no compensation occurs, and this channel determines the most of the total decay width of $\sigma_{\rm III}$.

Our calculations are rather qualitative. However, they allow us to conclude that $f_0(1500)$ is a scalar glueball state, whereas $f_0(1710)$ is a quarkonium, for the following reasons: 1) The total decay width of the glueball in our model is in better agreement with experiment if $f_0(1500)$ is assumed to be the glueball, rather than $f_0(1710)$. 2) As follows from our calculations, the main decay mode of the scalar glueball is that into four pions. This is true for the state $f_0(1500)$. A decay of $f_0(1710)$ into four pions, however, was not seen in experiment. 3) Moreover, a direct decay into a pair of ρ-mesons on their mass shell would be possible for $f_0(1710)$ if it was a scalar glueball. However, it was not seen. Our conclusion regarding $f_0(1710)$ as a quarkonium state is in agreement with the conclusion made in our papers [14]. Concerning the nature of $f_0(1500)$, we are in agreement with those in [15].

We are going to use the approach developed in our future work to describe all experimentally observed 19 scalar meson states that lie in the energy interval from 0.4 to 1.71 GeV. We hope to identify them with two scalar meson nonets (the ground and radially excited) and the scalar glueball ($f_0(1500)$).

We are grateful to Profs. A.A. Andrianov, D. Ebert and Drs. A.E. Dorokhov, S.B. Gerasimov, and N.I. Kochelev for useful discussions. The work is supported by RFBR Grant 00-02-17190.

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